NAG Toolbox for MATLAB

f07an

1 Purpose

f07an computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Syntax

```
[a, ipiv, b, info] = f07an(a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07an uses the LU decomposition with partial pivoting and row interchanges to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The by n coefficient matrix A.

2: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs_p** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv($*$) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

If **info** ≥ 0, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row **ipiv**(i). **ipiv**(i) $= i$ indicates a row interchange was not required.

3: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If **info** $= 0$, the n by r solution matrix X.

4: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **nrhs_p**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \|A^{-1}\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Following the use of f07an, f07av can be used to estimate the condition number of A and f07av can be used to obtain approximate error bounds. Alternatives to f07an, which return condition and error estimates directly are f04ca and f07ap.

8 Further Comments

The total number of floating-point operations is approximately $\frac{8}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this function is f07aa.

9 Example

```
a = [complex(-1.34, +2.55), complex(0.28, +3.17), complex(-6.39, -2.2),
complex(0.72, -0.92);
      complex(-0.17, -1.41), complex(3.31, -0.15), complex(-0.15, +1.34),
complex(1.29, +1.38);
     complex(-3.29, -2.39), complex(-1.91, +4.42), complex(-0.14, -1.35),
complex(1.72, +1.35);
     complex(2.41, +0.39), complex(-0.5600000000000001, +1.47), complex(-
0.83, ...
     -0.6899999999999999), complex(-1.96, +0.67)];
b = [complex(26.26, +51.78);
     complex(6.43, -8.68);
     complex(-5.75, +25.31);
     complex(1.16, +2.57)];
[aOut, ipiv, bOut, info] = f07an(a, b)
```

```
aOut =
   -3.2900 - 2.3900i   -1.9100 + 4.4200i   -0.1400 - 1.3500i    1.7200 +
1.3500i
    0.2376 + 0.2560i    4.8952 - 0.7114i   -0.4623 + 1.6966i    1.2269 +
0.6190i
   -0.1020 - 0.7010i   -0.6691 + 0.3689i   -5.1414 - 1.1300i    0.9983 +
0.3850i
   -0.5359 + 0.2707i   -0.2040 + 0.8601i    0.0082 + 0.1211i    0.1482 -
0.1252i
ipiv =
           3
           2
           3
           4
bOut =
   1.0000 + 1.0000i
   2.0000 - 3.0000i
  -4.0000 - 5.0000i
   0.0000 + 6.0000i
info =
           0
```